Vectors and Angles

The scalar product is useful for considering angles between vectors. This formula for 3D is in the formula booklet

2 dimensions	$\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \qquad \boldsymbol{w} = \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}$ $\boldsymbol{v} \cdot \boldsymbol{w} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \cdot \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = v_1 \cdot w_1 + v_2 \cdot w_2$
3 dimensions	$\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \qquad \boldsymbol{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$ $\boldsymbol{v} \cdot \boldsymbol{w} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \cdot \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = v_1 \cdot w_1 + v_2 \cdot w_2 + v_3 \cdot w_3$

Properties of Scalar Product

$$\boldsymbol{v} \cdot \boldsymbol{w} = \boldsymbol{w} \cdot \boldsymbol{v}$$
$$\boldsymbol{u} \cdot (\boldsymbol{v} + \boldsymbol{w}) = \boldsymbol{u} \cdot \boldsymbol{v} + \boldsymbol{u} \cdot \boldsymbol{w}$$
$$(k\boldsymbol{v}) \cdot \boldsymbol{w} = k(\boldsymbol{v} \cdot \boldsymbol{w})$$
$$\boldsymbol{v} \cdot \boldsymbol{v} = |\boldsymbol{v}|^2$$

Angle between 2 vectors ***v*** and ***w***

$$\boldsymbol{cos\theta} = \frac{\boldsymbol{v} \cdot \boldsymbol{w}}{|\boldsymbol{v}||\boldsymbol{w}|}$$

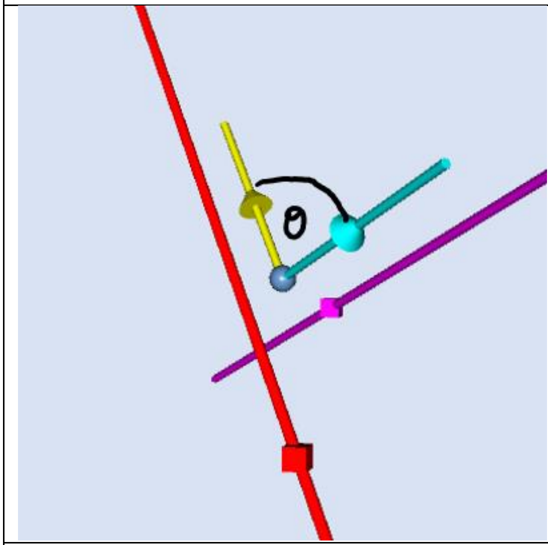
$|\boldsymbol{v}|$ is the magnitude of the vector ***v*** which we find using Pythagoras' Theorem

Useful Result

When 2 vectors are perpendicular

$$\boldsymbol{v} \cdot \boldsymbol{w} = 0$$

Angle between 2 Lines

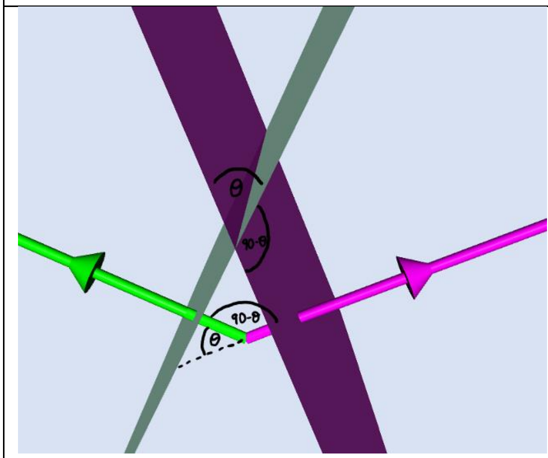


$L_1: \boldsymbol{r} = \begin{pmatrix}1\\0\\-2\end{pmatrix} + \lambda\begin{pmatrix}-1\\-1\\\sqrt{3}\end{pmatrix}$

$L_2: \mathbf{r} = \begin{pmatrix}-2\\-0.5\\-1\end{pmatrix} + \mu\begin{pmatrix}2\\2\\1\end{pmatrix}$

The angle between the two lines L_1 and L_2 is the angle between the vectors $\begin{pmatrix}-1\\-1\\\sqrt{3}\end{pmatrix}$ and $\begin{pmatrix}2\\2\\1\end{pmatrix}$

Angle between 2 Planes

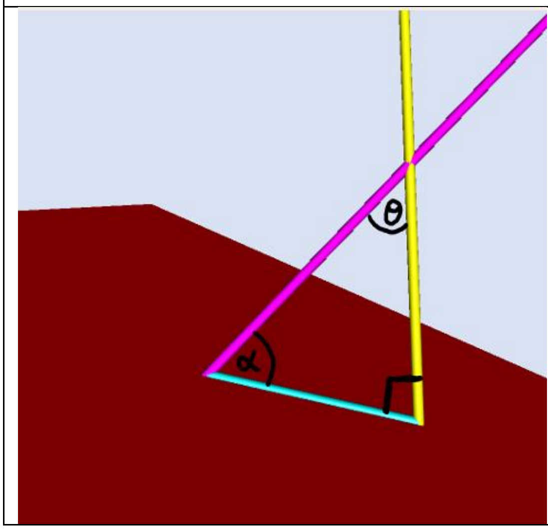


$\Pi_1 : 2x + 3y - 4z = 6$

$\Pi_2 : 1x - 1y + 2z = 2$

The angle between 2 planes $\Pi 1$ and Π_2 is the angle between the two normals $\begin{pmatrix}2\\3\\-4\end{pmatrix}$ and $\begin{pmatrix}1\\-1\\2\end{pmatrix}$

Angle between a Line and a Plane



$L : \mathbf{r} = \begin{pmatrix}0\\5\\4\end{pmatrix} + \lambda\begin{pmatrix}-1\\2\\0.5\end{pmatrix}$

$\Pi : 3x - 1y + 1z = 8$

Angle between line and plane = α

= angle between vectors $\begin{pmatrix}-1\\2\\0.5\end{pmatrix}$ and $\begin{pmatrix}3\\-1\\1\end{pmatrix}$

Angle between line and normal = θ

$\alpha = 90 - \theta$