

$$\mathbf{a} = 3\mathbf{i} + 2\mathbf{j} + \mathbf{k} \text{ and } \mathbf{b} = 2\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$$

Find a unit vector that is perpendicular to \mathbf{a} and \mathbf{b}

Find vector product

$$\begin{aligned} \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \times \begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix} &= \begin{pmatrix} 2 \times 2 - 1 \times 3 \\ -(3 \times 2 - 1 \times 2) \\ 3 \times 3 - 2 \times 2 \end{pmatrix} \\ &= \begin{pmatrix} 1 \\ -4 \\ 5 \end{pmatrix} \end{aligned}$$

$$\mathbf{v} \times \mathbf{w} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \times \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} v_2 w_3 - v_3 w_2 \\ v_3 w_1 - v_1 w_3 \\ v_1 w_2 - v_2 w_1 \end{pmatrix}$$

Check

$$\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -4 \\ 5 \end{pmatrix} = 3 - 8 + 5 = 0 \quad \begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -4 \\ 5 \end{pmatrix} = 2 - 12 + 10 = 0$$

Find magnitude of vector product

$$\begin{aligned} \left| \begin{pmatrix} 1 \\ -4 \\ 5 \end{pmatrix} \right| &= \sqrt{1^2 + (-4)^2 + 5^2} \\ &= \sqrt{1^2 + 16^2 + 25} \\ &= \sqrt{42} \end{aligned}$$

Unit vector

$$= \frac{1}{\sqrt{42}} \begin{pmatrix} 1 \\ -4 \\ 5 \end{pmatrix}$$