Let $cos\theta = \frac{2}{3}$, where $0 \leq \theta \leq \frac{\pi}{2}$

Find the value of

a) $sin\theta$

b) $sin2\theta$

c) $sin4\theta$

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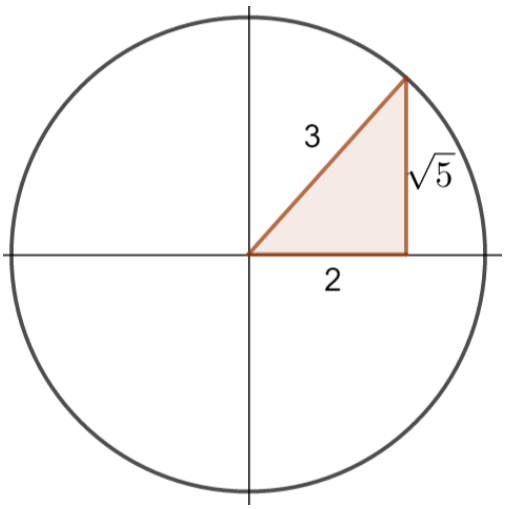
a)



$cos\theta = \frac{2}{3}$, where $0 \leq \theta \leq \frac{\pi}{2}$

We can find the opposite side in the right-angled triangle using Pythagoras' Theorem

$$sin\theta = \frac{\sqrt{5}}{3}$$

b) $sin2\theta \equiv 2sin\theta cos\theta$

$$sin2\theta = 2 \cdot \frac{\sqrt{5}}{3} \cdot \frac{2}{3}$$

$$sin2\theta = \frac{4\sqrt{5}}{9}$$

c) In order to find $sin4\theta$, we need to find $cos2\theta$

$$cos2\theta \equiv 2cos^2\theta - 1$$

$$cos2\theta = 2\left(\frac{2}{3}\right)^2 - 1$$

$$cos2\theta = -\frac{1}{9}$$

Notice, that since $cos2\theta$ is negative, then 2θ is an obtuse angle

$sin2\theta \equiv 2sin\theta cos\theta$

$sin4\theta \equiv 2sin2\theta cos2\theta$

$$sin4\theta \equiv 2\left(\frac{4\sqrt{5}}{9}\right)\left(-\frac{1}{9}\right)$$

$$sin4\theta \equiv -\frac{8\sqrt{5}}{81}$$