Consider the equation $8x^3 - 42x^2 + px - 27 = 0$.

a. State
   i. the sum of the roots of the equation
   ii. the product of the roots of the equation

b. The roots of this equation are three consecutive terms of a geometric sequence. Taking the roots to be $\frac{\alpha}{\beta}, \alpha, \alpha\beta$, show that one of the roots is $\frac{3}{2}$.

c. Solve the equation.

d. Find the value of p.

a.

$$8x^3 - 42x^2 + px - 27 = 0$$

Sum of roots $= \frac{42}{8} = \frac{21}{4}$

Product of roots $= \frac{27}{8}$

b.

$$\frac{\alpha}{\beta}, \alpha, \alpha\beta$$

Product of roots $= \frac{\alpha}{\beta} \times \alpha \times \alpha\beta$

$$\alpha^3 = \frac{27}{8}$$

$$\alpha = \frac{3}{2}$$

c.

Roots are $\frac{\frac{3}{2}}{\beta}, \frac{3}{2}, \frac{3}{2}\beta$

$$\frac{3}{2\beta}, \frac{3}{2}, \frac{3\beta}{2}$$

Sum of roots $= \frac{3}{2\beta} + \frac{3}{2} + \frac{3\beta}{2}$

$$\frac{3}{2\beta} + \frac{3}{2} + \frac{3\beta}{2} = \frac{21}{4}$$

$$\frac{3}{2\beta} + \frac{3\beta}{2} = \frac{15}{4}$$

$$\frac{3 + 3\beta^2}{2\beta} = \frac{15}{4}$$

$$6 + 6\beta^2 = 15\beta$$

$$6\beta^2 - 15\beta + 6 = 0$$

$$2\beta^2 - 5\beta + 2 = 0$$
$$(2\beta - 1)(\beta - 2) = 0$$
$$\beta = \frac{1}{2}, \beta = 2$$

Roots are $\frac{3}{4}, \frac{3}{2}, 3$

$$x = \frac{3}{4}, \frac{3}{2}, 3$$

d.

Polynomial equation $a(4x - 3)(2x - 3)(x - 3) = 0$

$$a(4x - 3)(2x^2 - 9x + 9) = 0$$
$$a(8x^3 - 36x^2 + 36x - 6x^2 + 27x - 27) = 0$$
$$a(8x^3 - 42x^2 + 63x - 27) = 0$$

$$8x^3 - 42x^2 + px - 27 = 0 \Rightarrow a = 1$$
$$p = 63$$