Find the coordinates of the **point of inflexion** on the curve $y = x^3 - 6x^2 + 13x - 9$

Non-stationary points of inflexion occur when

$$\frac{d^2y}{dx^2} = 0 \quad and \quad \frac{dy}{dx} \neq 0$$

$$y = x^3 - 6x^2 + 13x - 9$$

Find $\frac{dy}{dx}$

$$\frac{dy}{dx} = 3x^2 - 12x + 13$$

Find $\frac{d^2y}{dx^2}$

$$\frac{d^2y}{dx^2} = 6x - 12$$

Point of inflexion occurs when $\frac{d^2y}{dx^2} = 0$

Solve $\frac{d^2y}{dx^2} = 0$

$$6x - 12 = 0$$

$$6x = 12$$

$$x = 2$$

Check $\frac{dy}{dx}$ at this point

When $x = 2$, $\frac{dy}{dx} = 3(2)^2 - 12(2) + 13$

$$\frac{dy}{dx} = 12 - 24 + 13$$

$$\frac{dy}{dx} \neq 0$$

Hence, there is a non-stationary point of inflexion at $x = 2$

Find y coordinate

When $x = 2$, $y = (2)^3 - 6(2)^2 + 13(2) - 9$

$$y = 8 - 24 + 26 - 9$$

$$y = 1$$

There is a point of inflexion at (2,1)

In fact, we can show that there are no stationary points

Solve $\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = 3x^2 - 12x + 13$$

$$3x^2 - 12x + 13 = 0$$

Use quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{12 \pm \sqrt{(-12)^2 - 4(3)(13)}}{2(3)}$$

$$x = \frac{12 \pm \sqrt{144 - 156}}{6}$$

$$x = \frac{12 \pm \sqrt{-14}}{6}$$

The equation has no real roots

There are no stationary points