The 2nd, 3rd and 6th terms of an **arithmetic** sequence with common difference d, d≠0 form the first three terms of a **geometric** sequence, with common ratio, ***r***.
The 1st term of the arithmetic sequence is ***-2***.
a) Find ***d***.
The sum of the first ***n*** terms of the geometric sequence exceeds the sum of the first ***n*** terms of the arithmetic sequence by at least 1000.
b) Find the least value of ***n*** for which this occurs.

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a.

first term is -2

$$U_1 = -2$$
$$U_2 = -2 + d$$
$$U_3 = -2 + 2d$$
$$U_6 = -2 + 5d$$

Geometric sequence is $U_2,\ U_3,\ U_6$

$$-2 + d\,,\ -2 + 2d\,,\ -2 + 5d$$

$$r = \frac{U_2}{U_1}$$

$$r = \frac{-2 + 2d}{-2 + d}$$

$$r = \frac{U_3}{U_2}$$

$$r = \frac{-2 + 5d}{-2 + 2d}$$

$$\frac{-2 + 2d}{-2 + d} = \frac{-2 + 5d}{-2 + 2d}$$
$$(-2 + 2d)(-2 + 2d) = (-2 + 5d)(-2 + d)$$

Expand and simplify

$$4 - 8d + 4d^2 = 4 - 12d + 5d^2$$
$$0 = d^2 - 4d$$
$$0 = d(d - 4)$$
$$d = 0,\ d = 4$$

We know that $d \neq 0$

$$d = 4$$

Check by subsituting back into original terms

$$U_2 = -2 + 4 = 2$$
$$U_3 = -2 + 8 = 6$$
$$U_6 = -2 + 20 = 18$$

$$2\ ,\ 6\ ,\ 18$$

Geometric sequence with $r = 3$

b.

sum of the first ***n*** terms of the geometric sequence

$$S_n = \frac{U_1(r^n - 1)}{r - 1}$$
$$S_{n_g} = \frac{2(3^n - 1)}{3 - 1}$$
$$S_{n_g} = 3^n - 1$$

sum of the first ***n*** terms of the arithmetic sequence

$$S_n = \frac{n}{2}(2U_1 + (n-1)d)$$

$$S_{n_a} = \frac{n}{2}(2(-2) + (n-1) \times 4)$$

$$S_{n_a} = \frac{n}{2}(-4 + 4n - 4)$$

$$S_{n_a} = \frac{n}{2}(-8 + 4n)$$

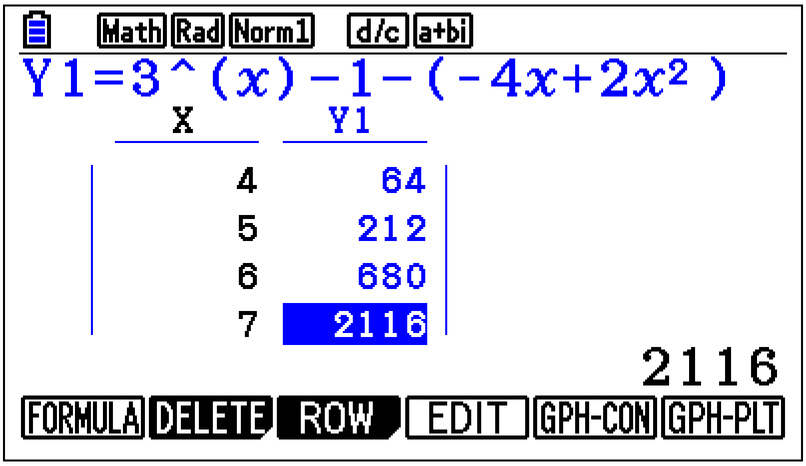
$$S_{n_a} = -4n + 2n^2$$

The sum of the first ***n*** terms of the geometric sequence exceeds the sum of the first ***n*** terms of the arithmetic sequence by at least 1000.

$$S_{n_g} - S_{n_a} > 1000$$

$$3^n - 1 - (-4n + 2n^2) > 1000$$

Solve using calculator



$$n = 7$$