Consider the geometric sequence where the first term is 45 and the second term is 36.

a) Find the least value of ***n*** such that the nth term of the sequence is less than 1
b) Find the least value of ***n*** such that the sum of the first ***n*** terms of the sequence is more than 200.
c) Find the sum to infinity.

a)

$U_1 = 45$
$U_2 = 36$

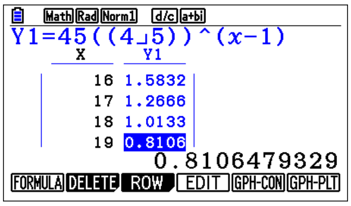
$$r = \frac{U_2}{U_1}$$
$$r = \frac{36}{45}$$
$$r = \frac{4}{5}$$

$$U_n = U_1 r^{n-1}$$
$$U_n = 45\left(\frac{4}{5}\right)^{n-1}$$

Find the least value of ***n*** such that the nth term of the sequence is less than 1

$$45\left(\frac{4}{5}\right)^{n-1} < 1$$

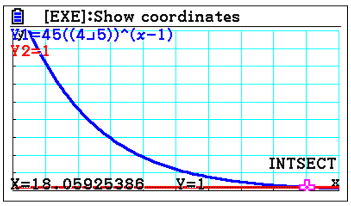
We can use the table function to solve this




$n = 19$

$U_{18} = 1.01$
$U_{19} = 0.811$

Or we can solve using a graph




$n = 19$

Or we can solve using logs

Solve

$$45\left(\frac{4}{5}\right)^{n-1} = 1$$

$$\left(\frac{4}{5}\right)^{n-1} = \frac{1}{45}$$

$$ln\left(\frac{4}{5}\right)^{n-1} = ln\frac{1}{45}$$

$$(n-1)ln\left(\frac{4}{5}\right) = ln\frac{1}{45}$$

$$n-1 = \frac{ln\frac{1}{45}}{ln\left(\frac{4}{5}\right)}$$

$$n-1 = 17.1$$

$$n = 18.1$$

$$n = 19$$

b)

$$S_n = \frac{U_n(1-r^n)}{1-r}$$

$$S_n = \frac{45\left(1-\left(\frac{4}{5}\right)^n\right)}{1-\frac{4}{5}}$$

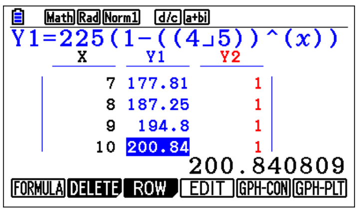
$$S_n = \frac{45\left(1-\left(\frac{4}{5}\right)^n\right)}{\frac{1}{5}}$$

$$S_n = 225\left(1-\left(\frac{4}{5}\right)^n\right)$$

Find the least value of ***n*** such that the sum of the first ***n*** terms of the sequence is more than 200.

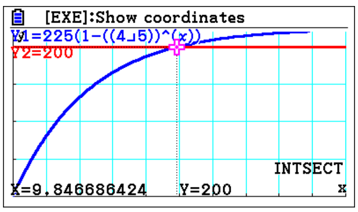
$$225\left(1-\left(\frac{4}{5}\right)^n\right) > 200$$

Use table




$n = 10$

Or graph




$n = 10$

Or by using logs

Solve

$$225\left(1-\left(\frac{4}{5}\right)^n\right)=200$$

$$1-\left(\frac{4}{5}\right)^n=\frac{200}{225}$$

$$\left(\frac{4}{5}\right)^n=\frac{20}{225}$$

$$ln\left(\frac{4}{5}\right)^n=ln\left(\frac{20}{225}\right)$$

$$n=\frac{ln\left(\frac{20}{225}\right)}{ln\left(\frac{4}{5}\right)}$$

$$n=9.85$$

$$n=10$$

c) Find the sum to infinity.

$$S_\infty=\frac{U_1}{1-r}$$

$$S_\infty=\frac{45}{1-\frac{4}{5}}$$

$$S_\infty=\frac{45}{\frac{1}{5}}$$

$$S_\infty=45\times 5=225$$