Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Higher level
Paper 1

24 October 2024

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

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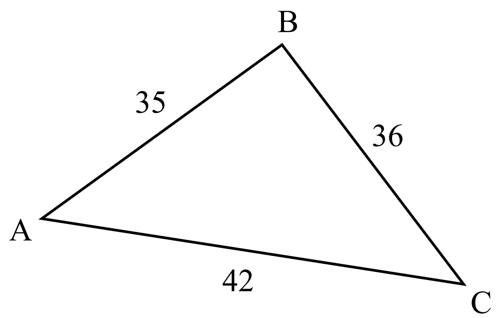
Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 5]

Consider the following triangle, ABC, such that $\mathrm{AB} = 35\,\mathrm{cm}$, $\mathrm{BC} = 36\,\mathrm{cm}$, and $\mathrm{CA} = 42\,\mathrm{cm}$.

diagram not to scale



(a) Find the value of $\mathrm{C\hat{A}B}$. [3]

(b) Find the area of the triangle ABC. [2]

2. [Maximum mark: 6]

Radioactive carbon is a material that decays over time.

The mass, $m(t)$ (in nanograms), of radioactive carbon in a fossil of a plant, after t years, can be modelled by the function

$$m(t) = 120e^{-0.000121t}$$

where t is the time since the plant died.

(a) Write down the initial mass of the radioactive carbon. [1]

(b) Find the mass of the radioactive carbon after $20\,000$ years. [2]

(c) Calculate the smallest number of complete years it takes for more than half the sample to decay. [3]

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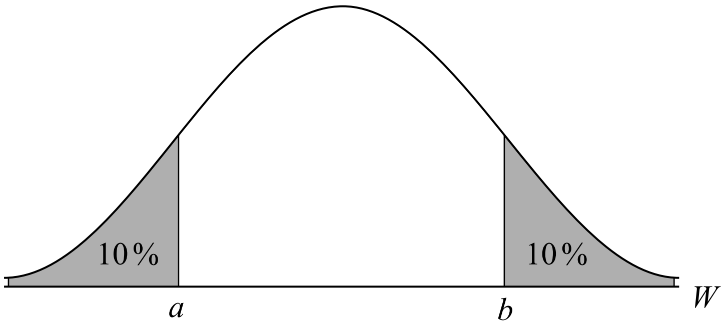
3. [Maximum mark: 7]

The mass, W, of Manx cats is normally distributed with a mean of $4.5\,\text{kg}$ and a standard deviation of $0.4\,\text{kg}$.

(a) A Manx cat is selected at random. Calculate the probability this cat's mass is more than $3.5\,\text{kg}$. [2]

The following curve represents this distribution. It is known that $\text{P}(W < a) = 0.1$ and $\text{P}(W > b) = 0.1$.



(b) Find the value of

(i) a

(ii) b. [3]

(c) Two Manx cats are selected at random from a large population. Find the probability that they both have a mass less than $3.5\,\text{kg}$. [2]

4. [Maximum mark: 5]

On 1 January in a particular year, Eva invests $\$25\,000$ in a new bank account. The account earns 5% simple interest, on the original $\$25\,000$, at the start of each subsequent year.

The amounts in the account at the start of each year form an arithmetic sequence.

(a) Find the common difference of this sequence. [2]

After k complete years, the amount in Eva's account will be greater than $\$44\,000$ for the first time.

(b) Find the value of k. [3]

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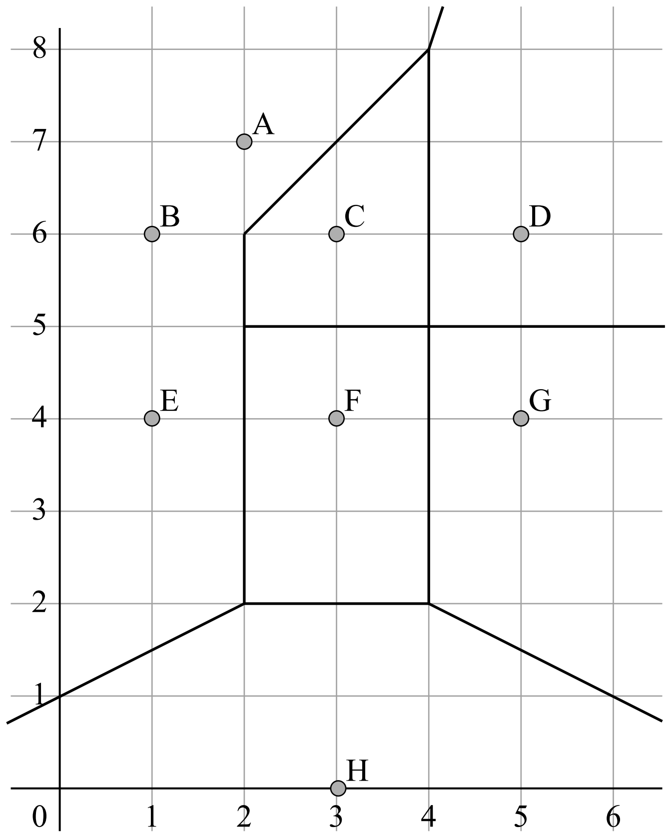
5. [Maximum mark: 6]

The sites in the Voronoi diagram represent eight hospitals in a city.



Each site has integer coordinates. Two edges are missing from the diagram.

(a) Draw the missing edges on the diagram. [2]

One square unit on the diagram represents $4\,\text{km}^2$ in the city.

(b) Find the area, in km^2, of the cell containing

(i) site F

(ii) site C. [3]

The hospitals at sites C and F have the same number of patients each year.

(c) Suggest a reason why the number of patients is not proportional to the area of the cell. [1]

(This question continues on the following page)

(Question 5 continued)

6. [Maximum mark: 4]

A museum has an annual membership fee of $\$200$, which includes 10 free visits. Any additional visits are charged at $\$30$ each. The total cost, $\$C$, of n visits during the year can be modelled by

$$C(n) = \begin{cases} 200, & n < p \\ an + b, & n \geq p \end{cases}, \text{ where } a, b, p, n \in \mathbb{Z}.$$

(a) Write down the value of

(i) a

(ii) p. [2]

(b) Find the value of b. [2]

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7. [Maximum mark: 6]

When Daniel retires, he invests $\$400\,000$ in an annuity fund that earns interest at a nominal rate of 4.5% per year, compounded monthly.

Daniel then withdraws $\$3600$ at the end of every month to pay for his living expenses.

(a) Find how much is in the annuity fund after 5 years. [3]

(b) Calculate how many times Daniel is able to make these withdrawals. [3]

8. [Maximum mark: 7]

The amount of daylight, L (in hours), in London in 2024 can be modelled by

$$L = a\sin\left(b(t-c)\right) + d,$$

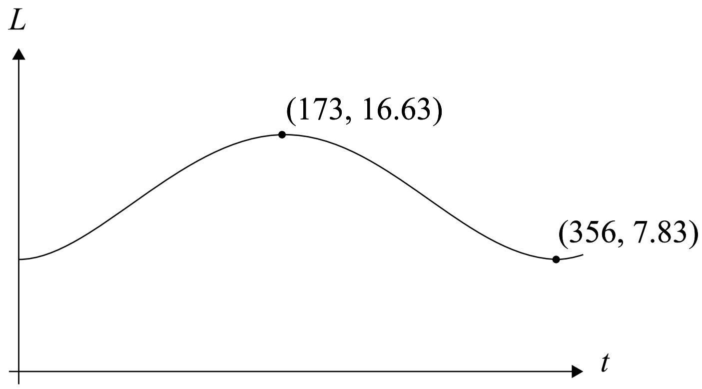
where $a, b, c, d > 0$ and t is the day of the year.

For example, day $1 =$ 1 January, day $2 =$ 2 January, and so on.

The maximum value of L is 16.63 hours on day 173 (21 June 2024).

The minimum value of L is 7.83 hours on day 356 (21 December 2024).

This information is shown in the following diagram.



Find the value of

(a) d [2]

(b) a [1]

(c) b [2]

(d) c. [2]

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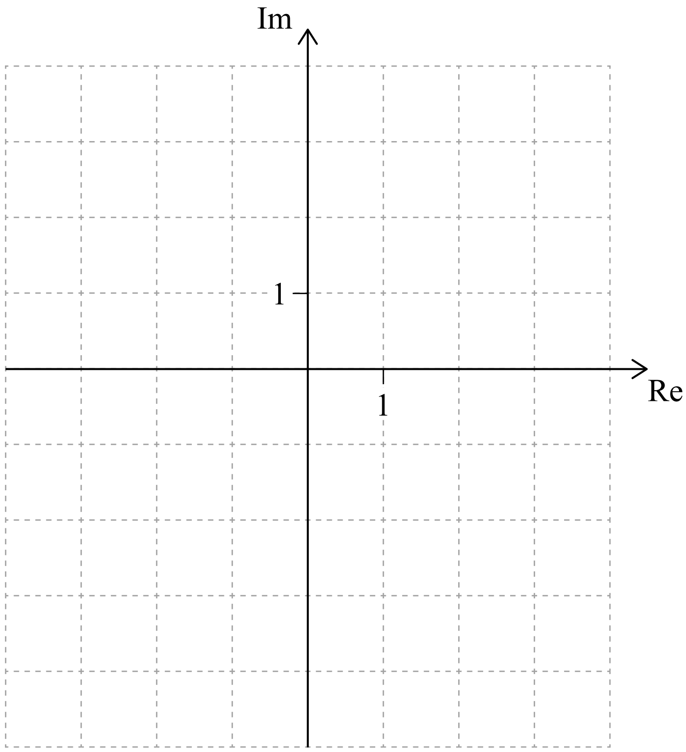
(Question 8 continued)

9. [Maximum mark: 5]

Let $z = 2 - 3\mathrm{i}$.

(a) Plot z on the Argand diagram. [1]



z can be written in the form $r \operatorname{cis}\theta$, where $r > 0$ and $-\pi < \theta \le \pi$.

(b) Find the value of

(i) r

(ii) θ. [2]

(c) Find the value of $z\mathrm{i}$ in the form $a + b\mathrm{i}$, where $a, b \in \mathbb{Z}$. [1]

$z\mathrm{i}$ can be obtained from z by a geometric transformation.

(d) By plotting $z\mathrm{i}$ on the Argand diagram, or otherwise, describe fully this transformation. [1]

(This question continues on the following page)

(Question 9 continued)

10. [Maximum mark: 8]

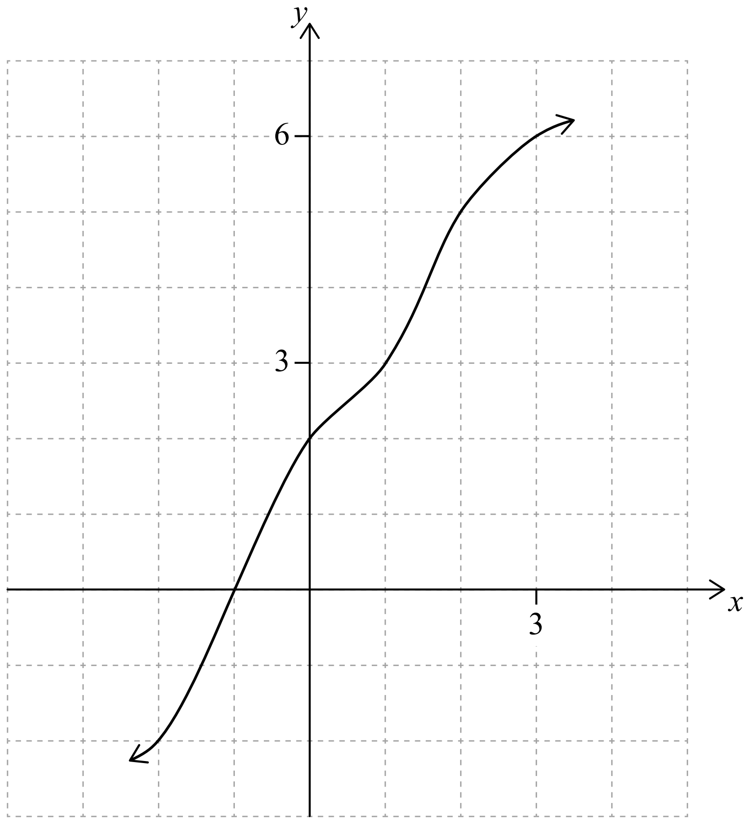
The function f is defined by $f(x) = 4\ln(2x-3)$, where $x > \frac{3}{2}$.

The graph of $y = f(x)$ is obtained from the graph of $y = \ln x$ by a sequence of three transformations.

(a) Describe fully these transformations, including the order in which they occur. [3]

The graph of $y = g(x)$ is shown.

The graph passes through the points $(-2, -2)$, $(-1, 0)$, $(0, 2)$, $(1, 3)$, $(2, 5)$ and $(3, 6)$.



(b) Find $(f \circ g)(2)$. [2]

(c) Solve $(f \circ g)(x) = 2\ln 9$. [3]

(This question continues on the following page)

(Question 10 continued)

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will not be marked.

11. [Maximum mark: 8]

Let $\boldsymbol{M} = \begin{pmatrix} -4 & 2 \\ -3 & 3 \end{pmatrix}$.

(a) Find the eigenvalues of $\boldsymbol{M}$. [3]

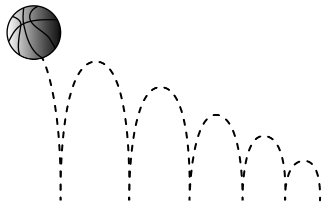
$\boldsymbol{M}$ can be written in the form $\boldsymbol{M} = \boldsymbol{PDP}^{-1}$, where $\boldsymbol{D}$ is a diagonal matrix.

(b) (i) Write down $\boldsymbol{D}$.

(ii) Find $\boldsymbol{P}$. [5]

12. [Maximum mark: 7]

Andreas drops a ball and records a video of the ball bouncing.



He uses the video to find the maximum height, in metres, after each of the first four bounces. His results are shown in the table.

Bounce number, n	Maximum height, h
1	0.613
2	0.514
3	0.439
4	0.377

Andreas thinks the maximum height can be modelled by the function

$$h(n) = 0.613\left(\frac{0.514}{0.613}\right)^{n-1}, \text{ where } n \in \mathbb{Z}^{+}.$$

(a) Complete the following table. [2]

Bounce number, n	Maximum height, h, according to the model
1	0.613
2	0.514
3	
4	

(b) Hence, calculate the sum of square residuals (SS_{res}). [2]

Andreas' friend thinks a better model for h could be found using an exponential least squares regression curve.

(c) (i) Find the equation of this model.

(ii) Use this model to estimate the height from which the ball was originally dropped. [3]

(This question continues on the following page)

(Question 12 continued)

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13. [Maximum mark: 6]

In 2008, there were no beavers in Scotland, as they had become extinct. In 2009, beavers were introduced from Norway. Surveys were carried out in the region of Tayside to monitor the beaver population.

Figure 1: A beaver

It is believed that the population, P, grows according to a logistic model

$$P = \frac{L}{1 + Ce^{-0.28t}},$$

where t is the number of years after 2008, $t \geq 1$.

A survey in 2017 estimated the population to be 433.

A survey in 2020 estimated the population to be 954.

Find the carrying capacity, L, for the number of beavers in Tayside, according to this model.

14. [Maximum mark: 8]

While playing on the beach, Sabine builds a mound of sand. The shape of the cross section through the centre of this mound can be modelled by the function

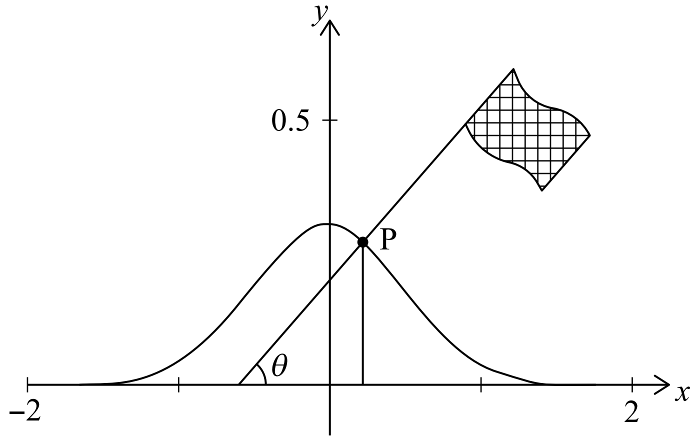
$$f(x) = 0.3\mathrm{e}^{-2x^2},$$

where $-2 < x < 2$.

Sabine places a flagpole into this mound at a point, P, which is $0.25\,\mathrm{m}$ above the level beach. The pole is perpendicular to the mound of sand at point P.

This information is shown in the diagram, where the x-axis represents the level beach and the y-axis represents the height above the beach. The scale on the axes is 1 unit $= 1$ metre.

diagram not to scale



The coordinates of P are $(a, 0.25)$, where $a > 0$.

(a) Calculate the value of a. [2]

(b) Find an expression for $f'(x)$. [2]

(c) Hence, or otherwise, find the angle, θ, the flagpole makes with the level beach. [4]

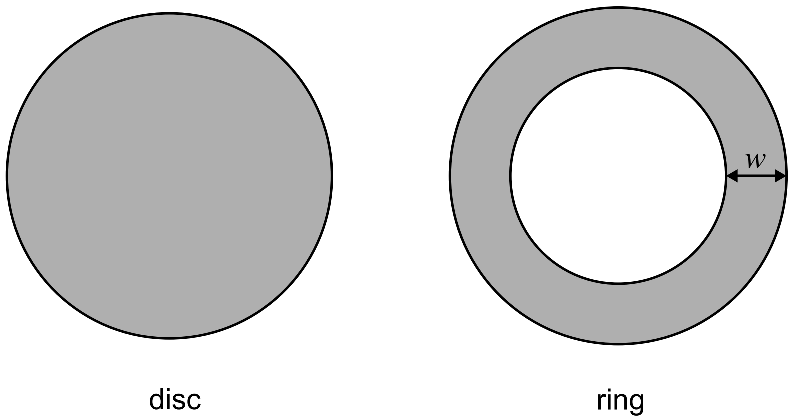
(This question continues on the following page)

(Question 14 continued)

15. [Maximum mark: 8]

A metal ring, of width w, is made by first cutting a circular disc and then cutting a circular hole exactly in the centre of this disc. This is shown in the diagram.

diagram not to scale



A machine produces many of these rings each day.

The diameters of the discs are normally distributed with mean 30 cm and standard deviation 0.8 cm.

(a) Find the distribution of the radii of the discs. [3]

The radii of the **holes** are normally distributed with mean 12 cm and standard deviation 0.25 cm.

(b) Calculate the probability that the width of a randomly chosen ring is less than 2.5 cm. [5]

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(Question 15 continued)

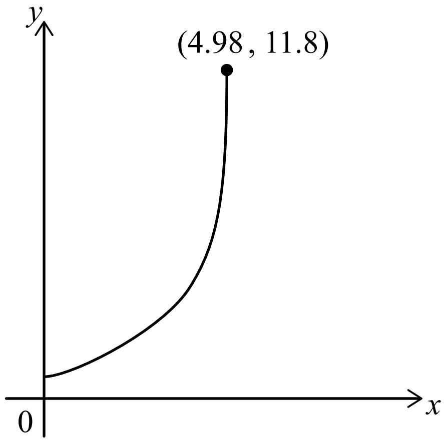
16. [Maximum mark: 7]

As part of his mathematics exploration, Jules models the shape of part of a wine glass to find the capacity (volume) of the glass.

He finds that the edge of half the glass can be modelled by the function

$$f(x) = 4 - 2\ln(5 - x), \text{ where } 0 \le x \le 4.98.$$

A graph of $y = f(x)$ is shown, with a scale of 1 unit $= 1\,\text{cm}$.



(a) Find the y-intercept. [1]

The point $(4.98, 11.8)$ represents a point at the top of the glass.

Let R be the region enclosed by the graph of f, the y-axis and the line $y = 11.8$.

Jules finds the capacity by rotating the region, R, $360°$ about the y-axis.

(b) Calculate the capacity of the glass that Jules obtains. [6]

(This question continues on the following page)

(Question 16 continued)

17. [Maximum mark: 7]

The fish in a lake feed on insects. Let F be the population of fish and L be the population of insects at time t (weeks).

The populations can be modelled using the coupled differential equations

$$\frac{\mathrm{d}F}{\mathrm{d}t} = 0.000004FL - 0.2F$$
$$\frac{\mathrm{d}L}{\mathrm{d}t} = 0.06L - 0.00003FL$$

where $F > 0$ and $L > 0$.

When $t = 0$, it is estimated that $F = 5000$ and $L = 80\,000$.

(a) For $0 \le t \le 52$, use Euler's method, with a step length $h = 4$, to find an approximate value for

(i) the maximum value of F

(ii) the minimum value of L. [5]

(b) (i) State what will happen to the number of fish in the lake, as predicted by the model.

(ii) Suggest a reason why this prediction may not occur. [2]

(This question continues on the following page)

(Question 17 continued)

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References:

Figure 1 animatedfunk, n.d. *Beaver Illustrations – stock illustration* [image online] Available at: https://www.gettyimages.co.uk/detail/illustration/beaver-illustrations-royalty-free-illustration/1453231718 [Accessed 30 November 2023]. Source adapted.

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